Midterm Exam for Practice

Instructions. You have 90 minutes to solve the questions. To get full credit you need to provide a detailed justification of your claims (a picture is not a proof!).

1. A consumer with income y buys x_1 units of good 1 at price p_1 and x_2 units of an input (at price p_2) that he uses to produce the second good z using a production function f, so that $z = f(x_2)$. This consumer pays income tax at a rate t. His problem is thus

$$\max_{x_1, x_2 \in \mathbb{R}^2_+} \{U(x_1, f(x_2)) \text{ s.t. } p_1 x_1 + p_2 x_2 = (1-t)\, y\}.$$

   Assume throughout that U and f are twice continuously differentiable.

   (a) Derive sufficient conditions for $\partial x_2(p, y)/\partial t < 0$. Make and justify natural assumptions on U and f to be able to use the Implicit Function Theorem to perform comparative statics. (25 points)

   (b) How does an increase in t affect the highest utility the consumer can achieve? (20 points)

2. Consider an expenditure minimizing consumer with a utility function for n goods $U(x)$ satisfying our standard assumptions. Show that if U is homogeneous of degree 1, then we must have $\partial x_i^h(p, u)/\partial u \geq 0$, for $i = 1, ..., n$. (You can invoke any result covered in class or in the HW's.) (25 points)

3. A factor of production is called inferior if its conditional factor demand decreases with output, i.e., $\partial x_i(w, y)/\partial y \leq 0$. Show that if marginal cost decreases as w_i increases, then factor i must be inferior. (30 points)

   Hint: marginal cost means $\partial C(w, y)/\partial y$.