Solution for Final Exam for Practice

1. We know that $\partial x_3(y,w)/\partial w_1 < 0$. By Young's theorem

$$\partial x_3(y,w)/\partial w_1 = \partial^2 C(y,w)/\partial w_3 \partial w_1 = \partial^2 C(y,w)/\partial w_1 \partial w_3 = \partial x_1(y,w)/\partial w_3 < 0.$$

   In addition, by the concavity of the cost function in w

$$\partial^2 C(y,w)/(\partial w_3)^2 = \partial x_3(y,w)/\partial w_3 \leq 0.$$

   By definition

$$f(x_1(y,w), x_2(y,w), x_3(y,w)) = y.$$

   Differentiting this expression with respect to w_3

$$f_1 \partial x_1(y,w)/\partial w_3 + f_2 \partial x_2(y,w)/\partial w_3 + f_3 \partial x_3(y,w)/\partial w_3 = 0.$$

   Thus, $\partial x_2(y,w)/\partial w_3 > 0$ is always true.

2. We know that there is an individual with utility function $u(.)$ who is concerned about monetary payoffs in the state of nature $s = 1, ..., S$ which may occur next period. Denote the dollar payoff in state s by x_s and the probability that state s will occur by p_s. The individual is assumed to choose $\mathbf{x} = (x_1, ..., x_S)$ so as to maximize the discounted expected value of the payoff. The discount factor is denoted by α. The set of feasible payoffs is denoted by X, which is assumed to be a non-empty, convex and compact sub-set of $\mathbb{R}^S$.

   (a) The problem of the consumer is

$$\max_{\mathbf{x}} \left\{ \sum_{s=1}^{S} \alpha u(x_s) p_s : \mathbf{x} \in X \right\}.$$

   (b) Let λ be an arbitrary constant.

$$\begin{aligned} V(\mathbf{p}, \lambda\alpha) &= \max_{\mathbf{x}} \left\{ \sum_{s=1}^{S} \lambda\alpha u(x_s) p_s : \mathbf{x} \in X \right\} \\ &= \lambda \max_{\mathbf{x}} \left\{ \sum_{s=1}^{S} \alpha u(x_s) p_s : \mathbf{x} \in X \right\} = \lambda V(\mathbf{p}, \alpha). \end{aligned}$$

   (c) Let $\mathbf{p}$ and $\mathbf{p}'$ be two arbitrary probability vectors, and $\lambda \in [0,1]$. Then

$$\begin{aligned} V(\lambda\mathbf{p} + (1-\lambda)\mathbf{p}', \alpha) &= \max_{\mathbf{x}} \left\{ \sum_{s=1}^{S} \alpha u(x_s)(\lambda p_s + (1-\lambda) p'_s) : \mathbf{x} \in X \right\} \\ &= \max_{\mathbf{x}} \left\{ \sum_{s=1}^{S} [\lambda\alpha u(x_s) p_s + (1-\lambda)\alpha u(x_s) p'_s] : \mathbf{x} \in X \right\} \\ &= \max_{\mathbf{x}} \left\{ \sum_{s=1}^{S} \lambda\alpha u(x_s) p_s + \sum_{s=1}^{S} (1-\lambda)\alpha u(x_s) p'_s : \mathbf{x} \in X \right\} \\ &\leq \max_{\mathbf{x}} \left\{ \sum_{s=1}^{S} \lambda\alpha u(x_s) p_s : \mathbf{x} \in X \right\} + \max_{\mathbf{x}} \left\{ \sum_{s=1}^{S} (1-\lambda)\alpha u(x_s) p'_s : \mathbf{x} \in X \right\} \\ &= \lambda \max_{\mathbf{x}} \left\{ \sum_{s=1}^{S} \alpha u(x_s) p_s : \mathbf{x} \in X \right\} + (1-\lambda) \max_{\mathbf{x}} \left\{ \sum_{s=1}^{S} \alpha u(x_s) p'_s : \mathbf{x} \in X \right\} \\ &= \lambda V(\mathbf{p}, \alpha) + (1-\lambda) V(\mathbf{p}', \alpha). \end{aligned}$$

3. We know that

$$\max_{y_1,y_2}\left\{\pi\left(y_1,y_2\right)=P_1\left(y_1\right)y_1+P_2\left(y_2\right)y_2-\alpha C\left(y_1+y_2\right)\right\}$$

The constraint sets are not affected by α. In addition,

$$\begin{aligned}\frac{\partial^2\pi\left(y_1,y_2\right)}{\partial y_1\partial\alpha} &= -C'\left(y_1+y_2\right)\leq 0\\ \frac{\partial^2\pi\left(y_1,y_2\right)}{\partial y_2\partial\alpha} &= -C'\left(y_1+y_2\right)\leq 0\\ \frac{\partial^2\pi\left(y_1,y_2\right)}{\partial y_1\partial y_2} &= -\alpha C''\left(y_1+y_2\right)\geq 0\end{aligned}$$

Thus, we need $C'\geq 0$ and $C''\leq 0$.