Final Exam for Practice

1. In the following statement if your answer is yes, prove it; if it is no, provide a counter example.

   Consider a cost-minimization problem with 3 inputs under the standard assumptions. Assume that $\partial x_3(y,w)/\partial w_1 < 0$, is it always true that $\partial x_2(y,w)/\partial w_3 > 0$? (30 points)

2. Consider an individual with utility function $u(.)$ who is concerned about monetary payoffs in the state of nature $s = 1, ..., S$ which may occur next period. Denote the dollar payoff in state s by x_s and the probability that state s will occur by p_s. The individual is assumed to choose $\mathbf{x} = (x_1, ..., x_S)$ so as to maximize the discounted expected utility of the payoff. The discount factor is denoted by α (i.e., $\alpha = 1/(1+r)$ where r is the discount rate). The set of feasible payoffs is denoted by X, which is assumed to be a non-empty, convex and compact sub-set of $\mathbb{R}^S$.

   (a) Write down the expected utility maximization problem. (15 points)

   (b) Define $V(\mathbf{p}, \alpha)$ to be the maximum discounted expected utility that the individual can achieve if the probabilities are $\mathbf{p} = (p_1, ..., p_S)$ and the discount factor is α. Show that $V(\mathbf{p}, \alpha)$ is homogenous of degree 1 in α. (25 points)

   (c) Show that $V(\mathbf{p}, \alpha)$ is a convex function of $\mathbf{p}$. (This point is not part of the practice test, but it is interesting!)

3. Suppose a firm with market power (a monopoly) sells its products in two different markets. Its problem is given by

   $$\max_{y_1, y_2} \{P_1(y_1)\, y_1 + P_2(y_2)\, y_2 - \alpha C(y_1 + y_2)\}$$

   where y_i is the level of production for market i, $P_i(.)$ is the inverse demand function for market i, $C(.)$ is the cost function, and $\alpha > 0$ is a cost-shifter. Using supermodularity, find conditions for $\overline{y}_1^*$ and $\overline{y}_2^*$ to be decreasing in α. (You can assume differentiability.) (30 points)