Final Exam: ECON 204A

Wednesday, 12/7/2016

Instructions. To get full credit you need to provide a detailed justification of your claims.

1. Matt is a utility-maximizer consumer with income level $y \in \mathbb{R}_{++}$ and a strictly increasing and strictly quasiconcave utility function $u : \mathbb{R}^n_+ \to \mathbb{R}$.

   (a) Suppose you know that Matt is indifferent between the two vectors of prices $\mathbf{p}, \mathbf{q} \in \mathbb{R}^n_{++}$. Is it true that Matt must prefer $\mathbf{p}$ to $(1/2)\,\mathbf{p} + (1/2)\,\mathbf{q}$? (Yes or No and justify your answer.) (15 points)

   (b) The government is considering two types of taxes for Matt. The first one is an income tax T and the second one is a per-unit tax t on good 1. You know that Matt will pay exactly the same amount of money to the government under the two possible taxes. Is it true that Matt must prefer the income tax to the per-unit tax? (Yes or No and justify your answer.) (10 points)

   (c) Show that if Matt's utility function $u(.)$ is homogenous of degree 1, then his indirect utility function can be written as $V(\mathbf{p}, y) = yV(\mathbf{p}, 1)$. (10 points)

2. Consider an agent with a utility function $u(w) = \ln(w)$. Starting with a wealth level w, he must decide what amount a of his wealth, with $0 \leq a \leq w$, to invest in a risky asset to maximize his expected utility. The random rate of return $\widetilde{r}$ of the risky asset is distributed as follows

$$\widetilde{r} = \begin{cases} 4c & \text{with probability } \frac{1}{2} \\ \\ -c & \text{with probability } \frac{1}{2} \end{cases}$$

   where $c > 0$ is just a constant.

   (a) Write down the maximization problem precisely. (10 points)

   (b) Find the optimal investment function $a^*(w)$, and show how $a^*(w)$ varies with w. (15 points)

   (c) Is the sign of $\partial a^*(w)/\partial w$ in part (b) surprising? (Justify your answer briefly and precisely) (10 points)

3. A profit maximizing firm produces one output by using two inputs K and L. Its production function $f(K, L)$ is twice continuously differentiable, strictly increasing and strictly concave. The prices of K and L are r and w, respectively. The price of the output is p. The government imposes a per-unit tax t on L.

   (a) Using Topkis' theorem, provide conditions under which the optimal levels of both inputs decrease in t. (15 points)

   (b) Assume the conditions you got in part (a) hold. Let us define

   $L^*(t, K) = \text{argmax}_L \{pf(K, L) - rK - (w + t)L\}$ and

   $K^*(t) = \text{argmax}_K \{pf(K, L^*(t, K)) - rK - (w + t)L^*(t, K)\}$

   Show that if $t' > t$ then $L^*(t', K^*(t)) \geq L^*(t', K^*(t'))$. (10 points)

   (c) Provide an economic interpretation of the result in part (b). (5 points)